Simultaneous Perturbation Stochastic Approximation-based Localization Algorithms for Mobile Devices

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Abstract—Localization precision remains active and open challenge in the area of wireless networks. For static network we develop model free approach of localization technique that bypasses the tedious modeling of diverse aspects to the contributing factor of localization errors, namely simultaneous perturbation stochastic approximation (SPSA) localization technique. The improved version of SPSA, simultaneous perturbation stochastic approximation by neighbor confidence (SPSA-NC) addresses error propagation of iterative localization controlled by incorporating a neighbor confidence matrix. The centralized SPSA and SPSA-NC does not scale well for the mobile environment due to the messaging requirements of repeated updates. We take distributed approaches to implement the aforementioned localization techniques for mobile devices by distributed simultaneous perturbation stochastic approximation (DSPSA) and distributed simultaneous perturbation stochastic approximation by neighbor confidence (DSPSA-NC) respectively; compare the results with the centroid (C) and weighted centroid (WC) localization techniques and show superiority of our methods.

Index Terms—Localization, mobile environment, constrained optimization, simultaneous perturbation stochastic approximation, neighbor confidence.

I. INTRODUCTION

Besides the importance of the location in relations to the sensed values, locations are utilized in various protocols and algorithms such as routing, clustering, topology control, coverage and others. Straightforward incorporation of GPS into all the devices is costly; GPS receivers require line-of-sight to GPS satellites; and finally GPS is unacceptably power hungry.

Localization techniques therefore evolve as alternate approaches to GPS. Here, a small subset of nodes with known locations (anchor nodes) along with the measured distances and/or angles are used to derive the unknown locations of the non-anchor nodes. Receive signal strength indicator (RSSI) [1], time of arrival (ToA) [2], time difference of arrival (TDoA) [3], and angle-of-arrival (AoA) [4] are the fundamental approaches to localizations in the literature.

Unfortunately each of the aforementioned approaches suffers from its own limitations. For example, RSSI-based estimation has high error in the derived distances contrarily AoA-based approach has reasonable accuracy with a higher device cost. Localization algorithm using the measurements requires be robust enough overcoming the adverse effect of the erroneous measurements. In practice localization algorithms commonly suffer from high localization errors where diverse aspects are liable for contributing to this error.

To address the limitations of the localization techniques we therefore develop a few variants localization techniques by utilizing a suitable optimization technique named simultaneous perturbation stochastic approximation (SPSA) [5] that minimizes the estimation error of approximation-based localizations.

Contributions: This paper presents distributed simultaneous perturbation stochastic approximation (DSPSA) and distributed simultaneous perturbation stochastic approximation by neighbor confidence (DSPSA-NC) attempting to address the issues: (i) minimizing estimation errors (ii) addressing flip ambiguity (iii) controlling error propagating in iterations and (iv) providing mobility support.

The rest of the paper organized as follows. Section II presents a background of localization algorithms related to our proposals. Section III gives the technical description of the developed localization methods. Sections IV and V present the simulation results and concludes the paper, respectively.

II. BACKGROUND

A large number of localization techniques have been proposed in the literature [6], [7], [8], [9], [10]. The following subsections briefs some of such localizations where our proposed algorithms provide improvements compared to the counterparts.

Minimizing Estimation Errors: RSSI-based localizations techniques such as maximum likelihood (ML) location estimation technique [11] and simulated annealing (SA)-based localization [12] utilize minimum mean square error (MMSE) and simulated annealing approach respectively to minimize location errors. ML suffers from poor localization accuracy for small neighbor set. Cross-Entropy (CE) localization technique [13], utilizes the cross-entropy method [14] in location estimation similar to SA with less computing power. Unfortunately both suffer from the well-known flip ambiguity.

Flip Ambiguity: Reference [15] attempts to address the flip ambiguity problem by quantifying the likelihood of flip through neighborhood geometric analysis. This analysis detects the flips and improves the error performance by eliminat-
ing the flips. Reference [16] handles the flip ambiguity problem by collinearity analysis. It also provides 3-connectivity of localizability to 2-connectivity by utilizing distance conflicts and eliminating ambiguity. Reference [17] proposes refinement techniques by correction operation where the underlying technique incorporates stochastic optimization through simulated annealing and genetic algorithm. A distributed variant of the solution is also provided with comparatively low accuracy. Our SPSA family of localizations handles the issue by constrained minimization technique utilizing penalty function method [18],[19].

**Error Propagation**: Localization by iterative least-square (ILS) provides error management for the least square location estimation technique [20], [21] where error is controlled by three basic criteria (i) error characterization on estimated locations (ii) neighbor selection and (iii) update criterion. SPSA-NC/DSPSA-NC rather avoids such modeling aspects and estimates the confidence of the neighbor nodes.

**Mobility Support**: A number of localization techniques are developed for static environment and centralized in nature where the location is estimated in the central location. SA, SPSA, SPSA-NC etc. are the examples of such static centralized localizations. In mobile environment the requirement of repetitively feeding the centralized node with measured angles and distances and updating the whole network with the derived locations becomes highly costly if not impractical. Our proposed DSPSA and DSPSA-NC handles such issue while preserving the strengths of the localization techniques of SPSA and SPSA-NC respectively.

**Training of Algorithm**: A number of localization algorithms find locations of the mobile devices using training phases [22], [23], [24], [25], [26]. We intentionally avoid such training due to its inconvenience even though such training based localization may provide good error performance in some cases.

### III. THE SPSA ALGORITHMS

This section deals with the SPSA family of localization algorithms consist of (i) fundamental SPSA and SPSA-NC localizations and (ii) customized and modified, mobile device-based algorithms DSPSA and DSPSA-NC techniques.

Let a set of $N$ total nodes deployed in the field defined as $n_1 : n_N$. And let the set of $A$ anchors defined as $n_{N-A} : n_N$ has their locations perfectly known and is a subset of the aforementioned set. The localization problem attempts to find the $[x_i, y_i]$ coordinates of node $n_i$ where $i = 1 : N - A$.

Fig. 1 depicts various functionalities of the SPSA family of localization. Where, Fig. 1(a) shows the virtual world on an interested node, Fig. 1(b) shows how a wrongly derived location is penalized and corrected, Fig. 1(c) shows the building blocks of the neighbor confidence and finally, Fig. 1(d) depicts the function block diagram of the SPSA family of localization techniques.

#### A. SPSA

Handling the localization problem commonly viewed as approximation problem of the unknown locations. The inputs to the approximation algorithm are mostly the neighbor distances and angles measured by hardware based on the signal strengths, directions and/or time difference of send-receive packets. A distance based localization algorithm calculates the distances from the approximated locations denoted as $d_{ij}$ compare them with the measured distances denoted as $d_{ij}$ and attempt to minimize the sum of differences between the two in an iterative fashion. Note that $i$ and $j$ represents the node $i$ and node $j$ in the neighborhood. The set of neighbors to node $i$ is denoted as $N_i$. Therefore the cost function of the approximation techniques can be represented as

\[
\tau = \sum_{j \in N_i} |d_{ij} - \hat{d}_{ij}|
\]

The aforementioned cost function suffers from two different types of limitations (i) cannot handle flips (ii) has no control on error propagation over rounds.

Note that flip occurs when a subset of nodes in the neighborhood is collinearly located. And a node or a set of node thereby wrongly estimated at the opposite side of the line. SPSA handles the flips by identifying the flips from the neighbor sets and penalize the flipped node. Penalty incorporated (i) if a neighbor node from the neighbor set finds the node estimated location is outside the neighbor radio range (ii) if a non-neighbor finds the node estimated location is inside the node radio range. The algorithm is such design that it pulls the wrongly estimated node location towards the neighborhood it belongs to as well pushing the node location outside the wrongly estimated neighborhood.

If set of neighbors and set of non-neighbors of node $i$ in the network are denoted as $N_i$ and $N_{\bar{i}}$. The transmission range of the nodes is denoted as $\bar{R}$. Therefore the optimization problem can be written as minimizing of

\[
\psi_i = \sum_{j \in N_i} (d_{ij} - \hat{d}_{ij})^2
\]

such that $d_{ip} \leq \bar{R} \ \forall p \in N_i$ and $\hat{d}_{iq} > \bar{R} \ \forall q \in N_{\bar{i}}$

Recall that the measured and estimated distances are denoted as $d_{ij}$ and $\hat{d}_{ij}$ respectively. Consequently the cost function can be expressed as Eq. (3) in the logarithmic penalized paradigm.

\[
\tau = \sum_{j \in N_i} |d_{ij} - \hat{d}_{ij}| + r_k \cdot \sum_{j \in (N_{\bar{i}} \cap N_{D_j})} |\ln(-|d_{ij} - \bar{R}|)|
+ r_k \cdot \sum_{j \in (ND_i \cap N_{\bar{i}})} |\ln(-|\hat{d}_{ij} - \bar{R}|)|
\]
Fig. 1. (a) Interested neighborhood of a node in the whole network. (b) Correcting forces on the flipped node by the neighborhood. (c) Deriving neighbor confidence (NC) from confidence on estimated locations (CEL) and confidence on measured distances (CMD). (d) Functional block diagram of the algorithms.

B. SPSA-NC

SPSA-NC, while incorporating the aforementioned penalty additionally handles the iterative error propagation by introducing a node confidence metric. Note that, the contributing factor of the error is twofold (i) error in distance measurement and (ii) error in estimated location. SPSA-NC therefore defines neighbor confidence matrix and handles both of them gracefully.

1) Deriving Neighbor Confidence: One of the most important problems with iterative localization algorithms is the error propagates in successive rounds. Here, we deal with well diversified and hard to model aspects of variable errors. The contributing factor is not limited to neighbor node type (anchor/non-anchor), hop distances from anchors, distances in meters from the nodes, radio range, % of anchor deployments, even the specific relative location patterns and wireless conditions. Utilizing the entire aforementioned criterion by modeling and incorporating makes the system rather extremely complex and impractical. Handling all the aforementioned criteria is handled by the neighbor confidence matrix in turn deals with input variables of SPSA engine.

Confidence on Estimated Locations (CEL): The relative measure of confidence provides the answer to the question how reliable the estimated location is. Simple modification on SPSA localization (a two dimensional estimation of \((x_i, y_i)\)) by making it a three dimensional approach in SPSA-NC makes the localization technique estimates the confidence on the estimated location and provides \(\psi_{\xi_i, x_i, y_i}\). Note that the confidence of an anchor node is always one as it is a true location.

Confidence on Measured Distances (CMD): Unlike CEL (belongs to each node) CMD belongs to each link and is measured as normalized inverse distance of two neighbors. \(\psi_{\zeta_{ij}}\) is therefore the confidence parameter measured at node \(i\) concerning neighbor node \(j\). In case of a flip a new pair may be created by the localization technique as the estimation of the distance becomes less than the radio range. With the incorporation of the penalty intern force the pair dismissed in rounds.

Neighbor Confidence (NC): NC provides answer to the question how much we believe the specific neighbor for its derived location and distance information while evaluating the current node location in this particular algorithmic run. Therefore in practice NC becomes is the normalized sum of the two aforementioned confidence measures, CEL and CMD. Being a function of a global and a local, parameter NC in
turn becomes a local parameter fed to the SPSA. Note a relative confidence factor \((rc)\) is introduced as weight to bridge between the two factors. Therefore the final NC can be derived by Eq. (4)

\[
\psi_{ij} = \frac{\psi_{i\max,j} \xi_{ij}}{\psi_{\xi,j}} + rc \cdot \frac{|\hat{d}_{ij} - d_{ij}|}{\psi_{i\max,j}}
\]

(4)

**C. Distributed SPSA (DSPSA) and Distributed SPSA-NC (DSPSA-NC)**

The superiority of the SPSA and SPSA-NC is restricted to the static sensor networks as they require all the distance and anchor information of the network. In mobile environment as the relative distances of the nodes are continuously changing it becomes costly and impractical to update information continuously to the whole network.

We attempt to retain the strength of the localization techniques of SPSA and SPSA-NC and modify them to a distributed approach cope with the mobile environment by limiting the information flow up to two-hop neighborhood. Nodes refresh the neighbors with its information with some predefined intervals upon receiving the updated information. Alternately nodes maintain their own world only up to two hops makes it scalable for the mobile environment.

**D. Localization Techniques**

The SPSA-based iterative localization techniques start with randomly initializing locations in the \([x, y]\) coordinates of all nodes in the network. Algorithms randomly select a single node of interest at that particular run and calls SPSA optimization engine for approximating the node location based on the neighbor distances. Localization algorithm then selects a second node upon updating the location information from SPSA engine. The iteration continues until the correction becomes less than a predefined threshold. The threshold should set to reasonably small to get a suitable result.

**Optimization Engine:** We take the simultaneous perturbation stochastic optimization as a tool to minimize the estimated locations of the non-anchor nodes in the field. The core strength of the tool is it does not require modeling the gradient of the cost function. Note that modeling for handling the different contributing factors are tedious in localization perspective due to a large number of contributing factors. Even though a gradient estimation is not required in this method, the performance remains as good as the gradient-based approaches. And finally, SPSA optimization is capable of handling erroneous measurements. Note that, common distance measuring techniques (signal strength-based, time of arrival-based and time difference of arrival cases) always delivers erroneous measurements.

**Optimization Procedure:** The fundamental problem to the core optimization technique is to minimize the differentiable cost functions defined earlier. The algorithms utilize a general recursive procedure to estimate the desired parameter (location and/or confidence in this specific case) by iterative random perturbations and gradient estimations. The perturbations and gradient estimations are detailed in [5], [18], [19].

**Notes:** Even with the limited world of each node the algorithm DSPSA and DSPSA-NC are not suitable for implementing in tiny sensors. Rather our attempt is to utilize the strength of SPSA and SPSA-NC-based static sensor network localization into smart phones like high end mobile devices.

**Common localization for such network currently often depends on the training-based approach and performs well. Training often involves human intervention therefore must be avoided. Even with the automated training mechanisms it may take a reasonably long time to train the system to an acceptable level.**

**IV. Simulation Results**

We simulate our localization algorithms in Matlab. We compare our DSPSA and DSPSA-NC with centroid (C) and weighted centroid (WC) localization techniques. Note that in C, the unknown location of the node is calculated by the centroid of all the neighboring nodes. In WC a weight is incorporated and a weighted centroid measure is taken. Note that the weight is inversely proportional to the squared distance of the particular neighbor.

We set 100m × 100m filed with 100 nodes. Among them \(n\) number of fixed anchor nodes in grids and the rest of the nodes are mobile. Here, \(n = 9, 16, \) or 25. We analyze our DSPSA and DSPSA-NC through random waypoint mobility model [27]. Note that the random waypoint mobility model attempts to model the mobile user. It defines the location, velocity and acceleration of mobile devices that change over time randomly and independently of other nodes. Walk time and pause time are set as 2 – 6 sec and 0 – 1 sec respectively. The speed is varied from \(v_{\text{min}} \rightarrow v_{\text{max}}.\)

The error is defined as Eq. (5) where \(N\) and \(A\) are denoted as total number of nodes and number of anchors respectively. And where \((x_i, y_i)\) and \((\hat{x}_i, \hat{y}_i)\) are denoted as the true and estimated locations of node \(i\) respectively.

\[
\text{error} = \frac{1}{N - A} \sum_{i=A+1}^{N} \frac{(x_i - \hat{x}_i)^2 + (y_i - \hat{y}_i)^2}{R^2}
\]

(5)

Fig. 2 presents the instantaneous error of the location estimation of the interested techniques for 100 seconds with different anchor deployments. Here, \(v_{\text{min}} = 3\) and \(v_{\text{max}} = 10.\)

In the beginning of the algorithmic runs instantaneous error is quite high and decreases sharply. In this region the algorithms converges, where DSPSA and DSPSA-NC converges in less time i.e., with less number of runs. In the successive figures the error measurement is taken discarding the first exponentially decaying region i.e., error in the initial phase is neglected. And for all the cases RMS errors are taken from the instantaneous measurements. Such five runs are then averaged by using the RMS again.

Fig. 3 depicts the error performance of the algorithms with different speed of the nodes. We set \(v_{\text{max}} = 10\) where the \(v_{\text{min}}\) is varied from 1 to 10. Naturally with increasing velocity the error increases for all the cases. Weighted centroid (WC) was introduced to improve the error performance of
the localization of the centroid (C) algorithm. Our interesting observation is WC performs better than C only in low velocity scenario. On the other hand simpler C outperforms WC in high velocity scenario. Our DSPSA and DSPSA-NC perform way better than both C and WC approach in both low and high velocity scenarios.

Fig. 4 provides the performance of the algorithm with different noise factors (20, 40, 60, 80, and 100). To overcome the impact of the velocity and variance of the velocity we set the $v_{\min} = v_{\max} = 3$. In this case C performs the worst followed by WC. Increasing NF has worst impact on WC where C is quite stable to different NF. DSPSA-NC performs the best with exception in 25 anchor case where DSPSA rather performs better. The impact of NF compared to the velocity on the algorithms is quite low when observing the slopes of Fig. 3 vs. Fig. 4.

V. CONCLUSIONS AND FUTURE WORK

The simultaneous perturbation stochastic approximation technique is utilized to minimize the location error in the wireless networks in SPSA localization. Radio range-based constraint is devised in the localization technique by penalty function method subsequently addresses the flip ambiguity problem. Controlling the error propagating from neighbors is addressed by utilizing a neighbor confidence matrix in SPSA-NC. The aforementioned SPSA and SPSA-NC is mostly suitable for the static environment as they are centralized localization technique. Retaining the strengths of the protocols and taking them to the mobile environment is achieved by reshaping the protocols into distributed localization techniques as DSPSA and DSPSA-NC. The error performance of the protocols is compared with C and WC methods of localization in random waypoint mobility model with different settings of velocities and noise factors. In all the cases our distributed methods of localization algorithms outperforms the C and WC methods thereby justifies our development effort.

A number of sensors are generally built-in into the mobile devices these days. Exploiting the capabilities of the other measurement like sound, light intensity and movements (through accelerometers and gyroscopes) along with the radio ranges can improve the localization accuracy further and is our future direction of research.

ACKNOWLEDGMENT

This research was sponsored by the Government of Abu Dhabi, United Arab Emirates through its funding of Masdar Institute’s research project on “Monitoring and Optimization of Renewable Energy Generation using Wireless Sensor Data Analytics”.
Fig. 4. Localization error (RMS) vs. noise factor.

(a) Anchors: 9, 16 and 25.

(b) In 3D.

REFERENCES


